DOCUMENT RESUME

ED 296 872 SE 049 229

AUTHOR Hunter, Walter M.
TITLE The Axial Curve Rotator.
INSTITUTION Mathematics Education Trust, Reston, VA.
PUB DATE 87
NOTE 34p.; Drawings and photographs may not reproduce well.
PUB TYPE Guides - Classroom Use - Guides (For Teachers) (052)

EDRS PRICE MF01/PC02 Plus Postage.
DESCRIPTORS Analytic Geometry; *Calculus; Graphs; *Manipulative Materials; Mathematical Concepts; Mathematics Curriculum; Mathematics Education; *Mathematics Instruction; Secondary Education; *Secondary School Mathematics; *Volume (Mathematics)
IDENTIFIERS *Axial Curve Rotator

ABSTRACT

This document contains detailed directions for constructing a device that mechanically produces the three-dimensional shape resulting from the rotation of any algebraic line or curve around either axis on the coordinate plant. The device was developed in response to student difficulty in visualizing, and thus grasping the mathematical principles associated with curve rotation around the x- or y-axis. In addition to the directions, the document also includes a discussion of the development of prerequisite concepts, a lesson description, photographs of the axial curve rotator, and real world applications of the teaching unit. (PK)

****
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
****

ED296872

MATHEMATICS EDUCATION TRUST

THE AXIAL CURVE ROTATOR

"PERMISSION TO REPRODUCE THIS MATERIAL HAS BEEN GRANTED BY
Cynthia C. Rosso

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)."

U.S. DEPARTMENT OF EDUCATION
Office of Educational Research and Improvement
EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

☑ This document has been reproduced as received from the person or organization originating it.
☐ Minor changes have been made to improve reproduction quality.

• Points of view or opinions stated in this document do not necessarily represent official OERI position or policy.

Walter M. Hunter
Jacksonville High School

SE 049 229

MATHEMATICS EDUCATION TRUST

THE AXIAL CURVE ROTATOR

Walter M. Hunter
Jacksonville High School

These materials have been produced with financial support from the Mathematics Education Trust. Any opinions, findings, and conclusions or recommendations expressed are those of the writers and do not necessarily reflect the views of the Mathematics Education Trust.

Copyright © 1987 by
The Mathematics Education Trust
1906 Association Drive
Reston, Virginia 22091

Printed in the United States of America

Permission to photocopy material from this document is granted to classroom teachers for instructional use. Permission must be sought for commerical use of content.

1. Project Description

In helping many students during the past few years with daily assignments from their calculus class, I have noticed one topic in particular that students have difficulty visualizing, and thus grasping its mathematical principles--curve rotation around the x- or y-axis. Textbooks, as well as the mind, work well in two dimensions, but the required visualizations in three dimensions, which are necessary for curve rotation, are extremely difficult for most students to master. As a result of this difficulty in student comprehension, I have designed and constructed a device that mechanically produces the three-dimensional shape resulting from the rotation of any algebraic line or curve around either axis on the coordinate plane--the Axial Curve Rotator.

After students observe and use the Axial Curve Rotator in class, a difficult concept from the calculus curriculum is simplified and brought to life. As a result, the students find the textbook explanation of the principles of curve rotation much easier to understand.

2. The Curve Rotator--Description and Diagrams

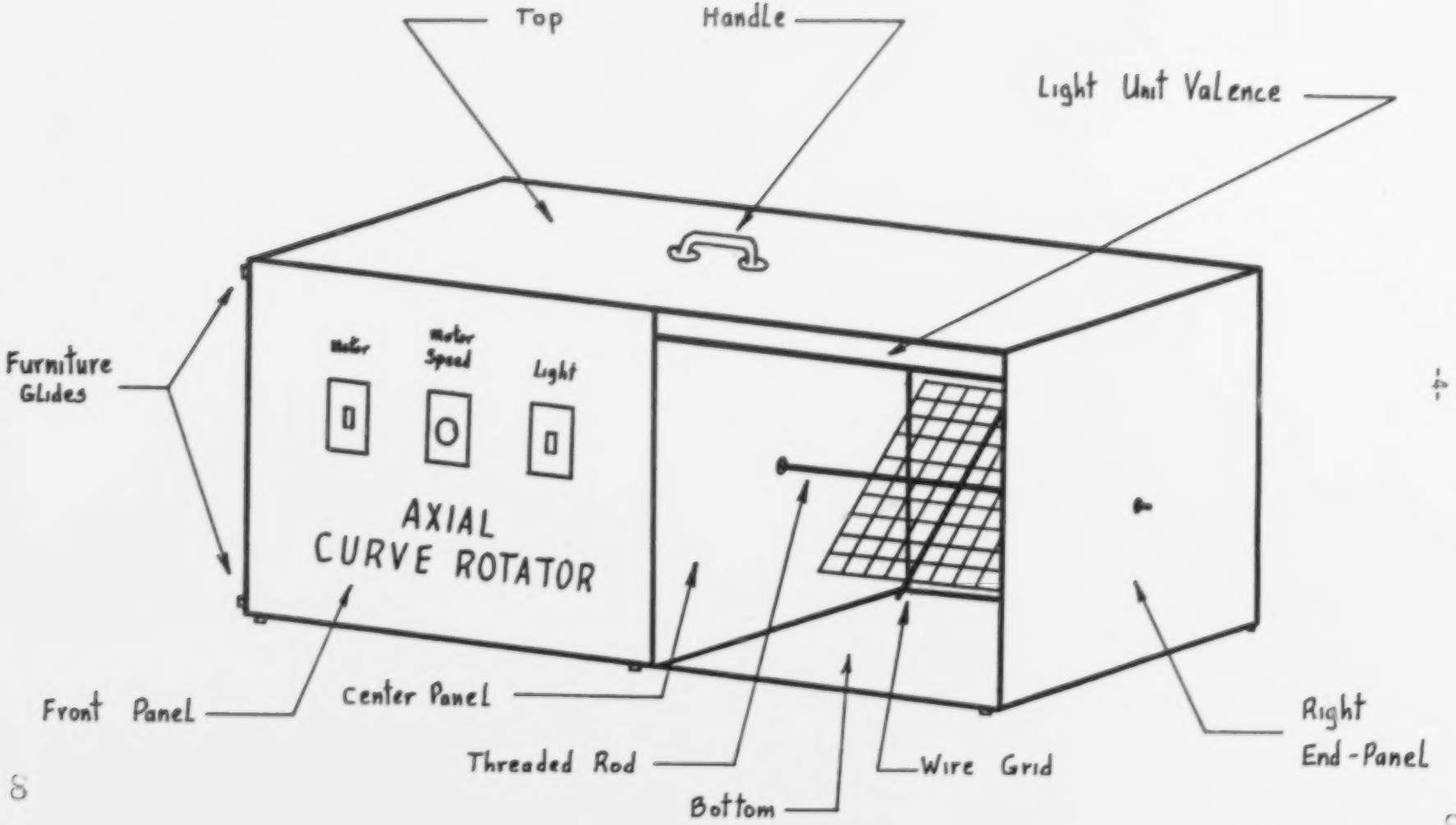
The Axial Curve Rotator is an electrically driven mechanical device whose wire grid's rate of rotation is controlled by a dimmer-switch rheostat. The rotating grid was constructed by first cutting the bottoms from two legal-sized letter trays, then welding them to a 22-inch length of threaded rod. The threaded rod's diameter was selected to match the size of the motor shaft. During assembly, the threaded rod was coupled to the motor shaft by tightening two pipe clamps onto a semicircular cap that bridged from the threaded rod onto the motor shaft. Thus as the motor shaft rotates, so do the attached threaded rod and grid.

When the unit is placed in its normal horizontal position, the motor shaft and threaded rod alignment will produce x-axis rotation. If y-axis rotation is desired, rotate the unit in a counterclockwise direction by pulling the handle atop the unit. The threaded rod at the center of the wire grid, along with the motor shaft, is now oriented vertically. When the unit is in either position, assume the x-axis is in its customary left-right direction and the y-axis is vertical.

The entire unit was painted flat black. All light, both external and internal, is absorbed by the black paint, except for the light coming from the white pipe cleaner attached to the grid. Since the only light being reflected comes from the white pipe cleaner, the algebraic curve stands out against the black background.

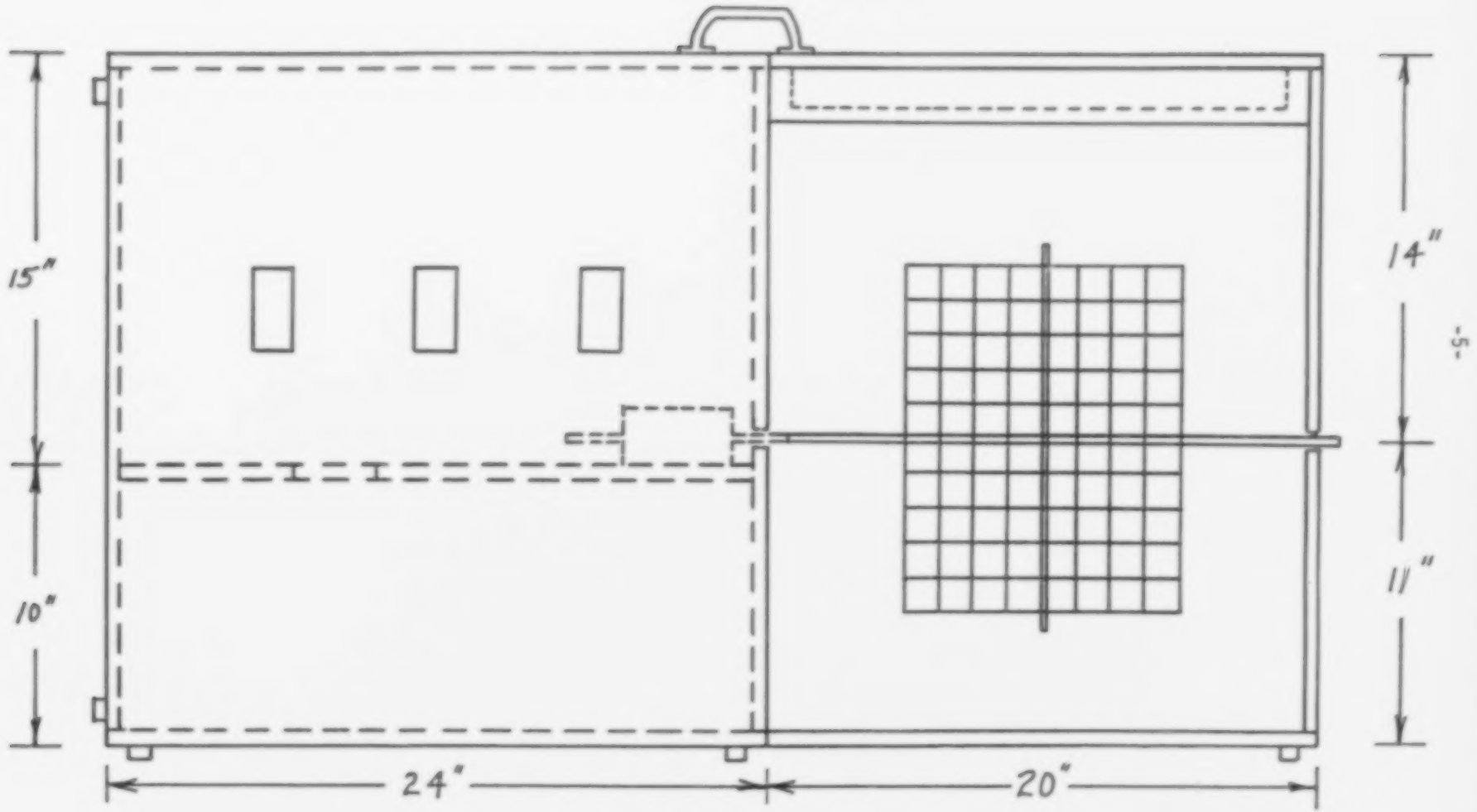
Once the white pipe cleaner is attached to the grid, the fluorescent light unit should be turned on. The motor switch is then turned on, followed by the dimmer-switch rheostat. As the electrical current passing through the rheostat slowly increases, the grid begins rotating

faster and faster, until the simple algebraic curve is transformed into a three-dimensional shape. Two-dimensional algebra can now be brought to life as three-dimensional calculus curve rotation.

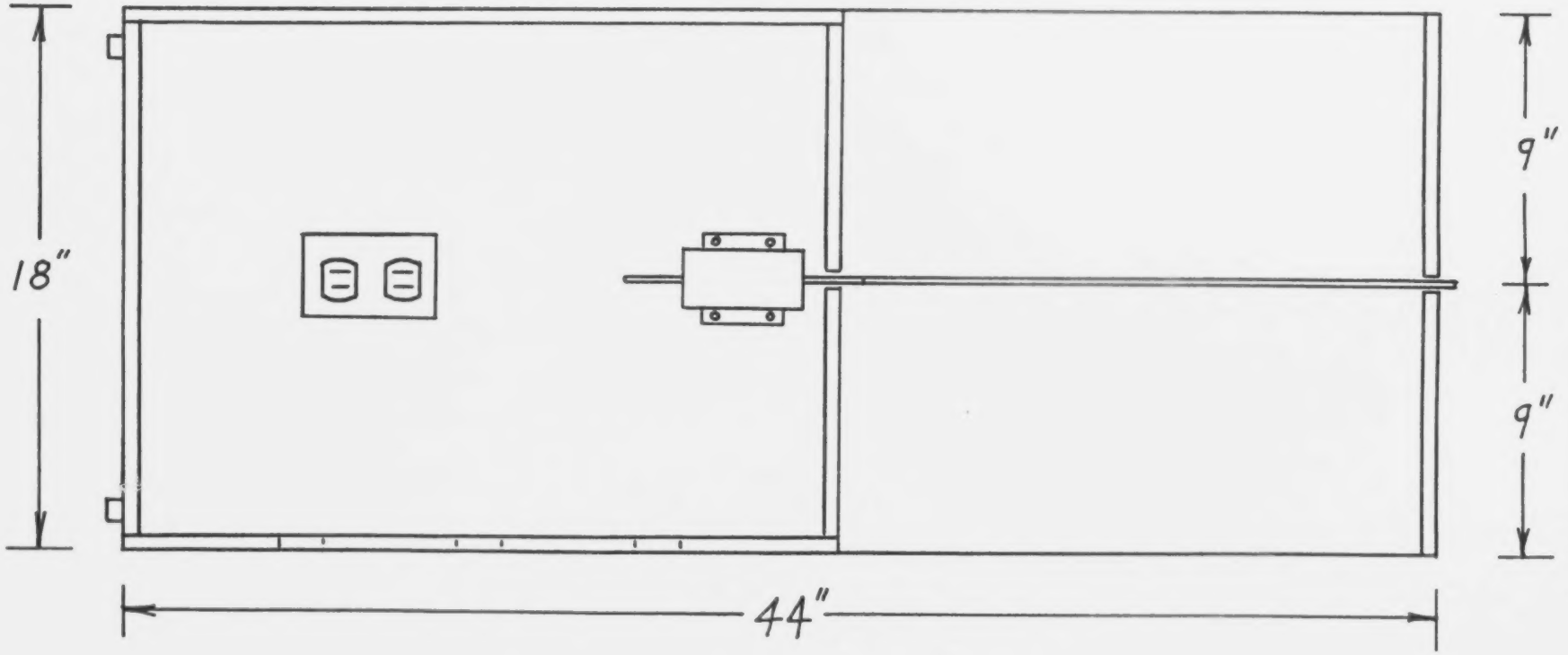

Top
Handle
Light Unit Valence
Furniture Glides
Motor
Motor Speed
Light
AXIAL CURVE ROTATOR
Front Panel
Center Panel
Threaded Rod
Bottom
Wire Grid
Right End-Panel

Front View

Scale : ¼" = 1"

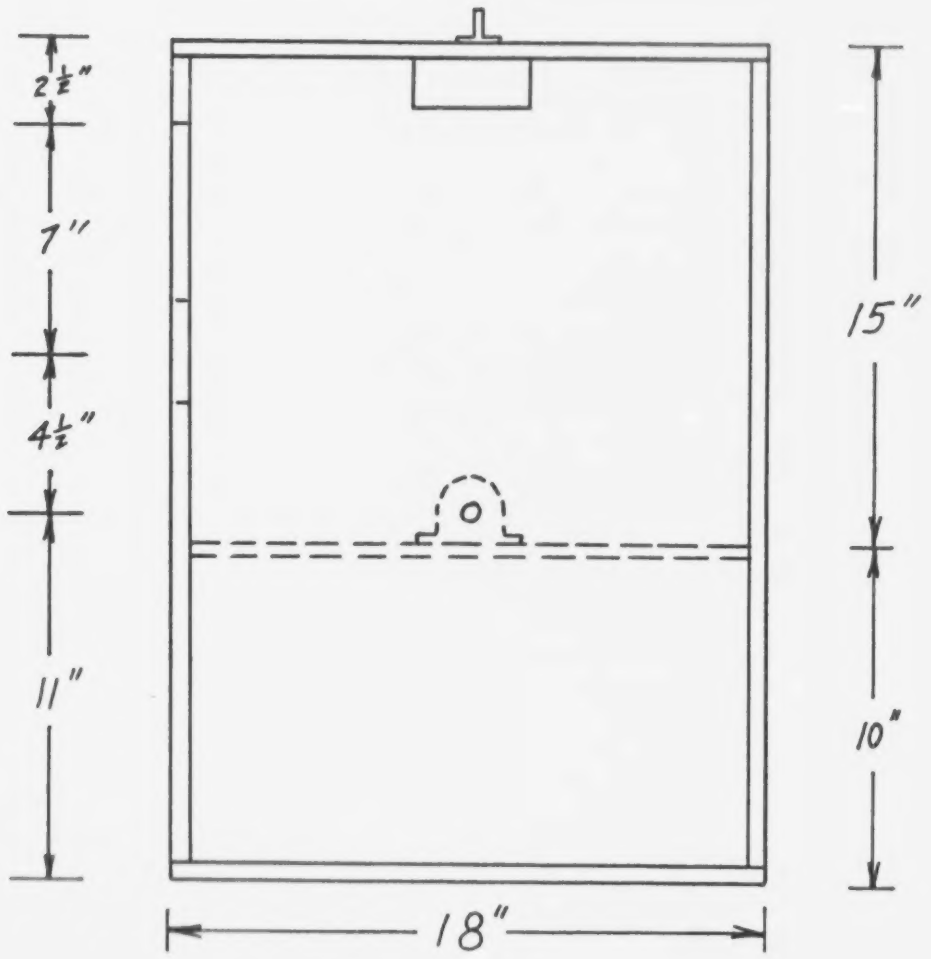


Top View of Unit (w/o Top)



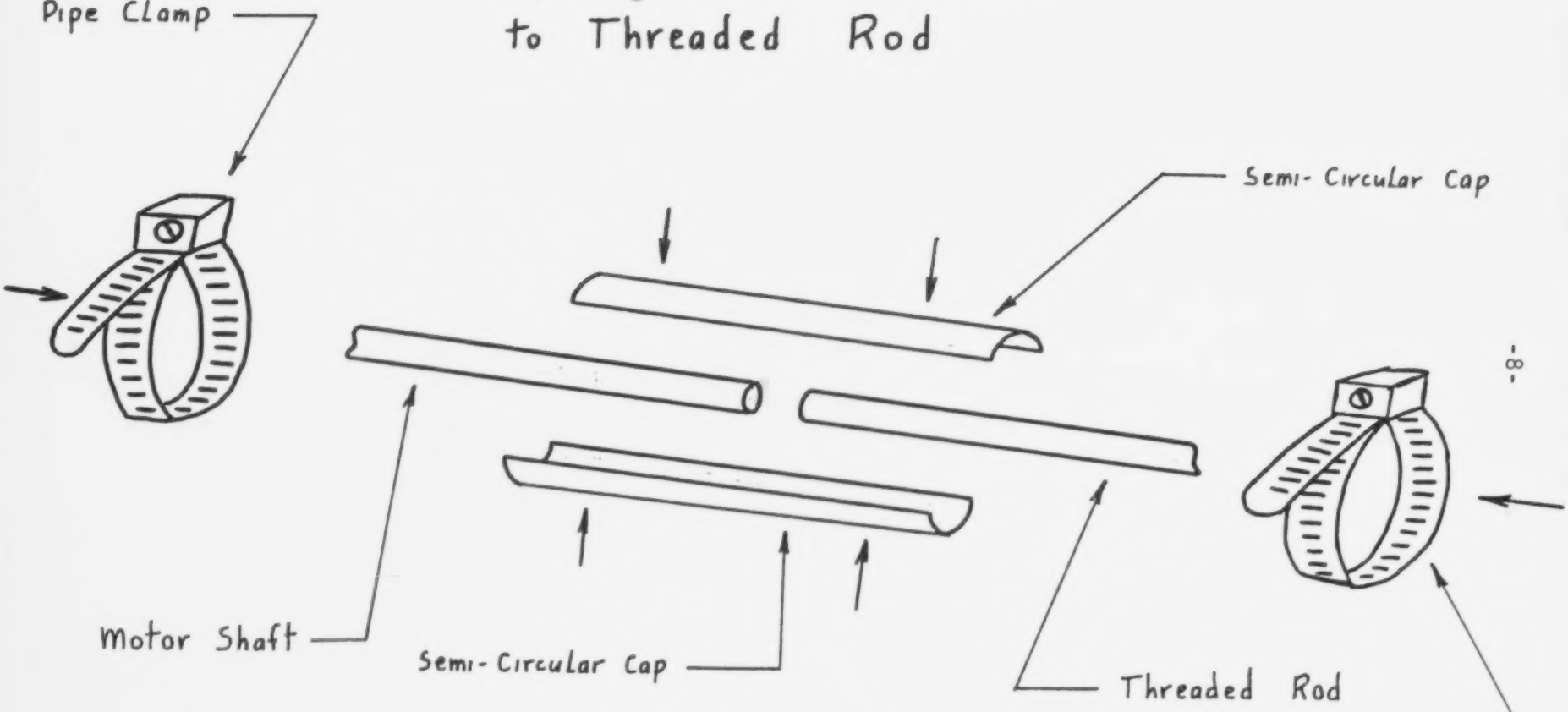
Scale : ¼" = 1"

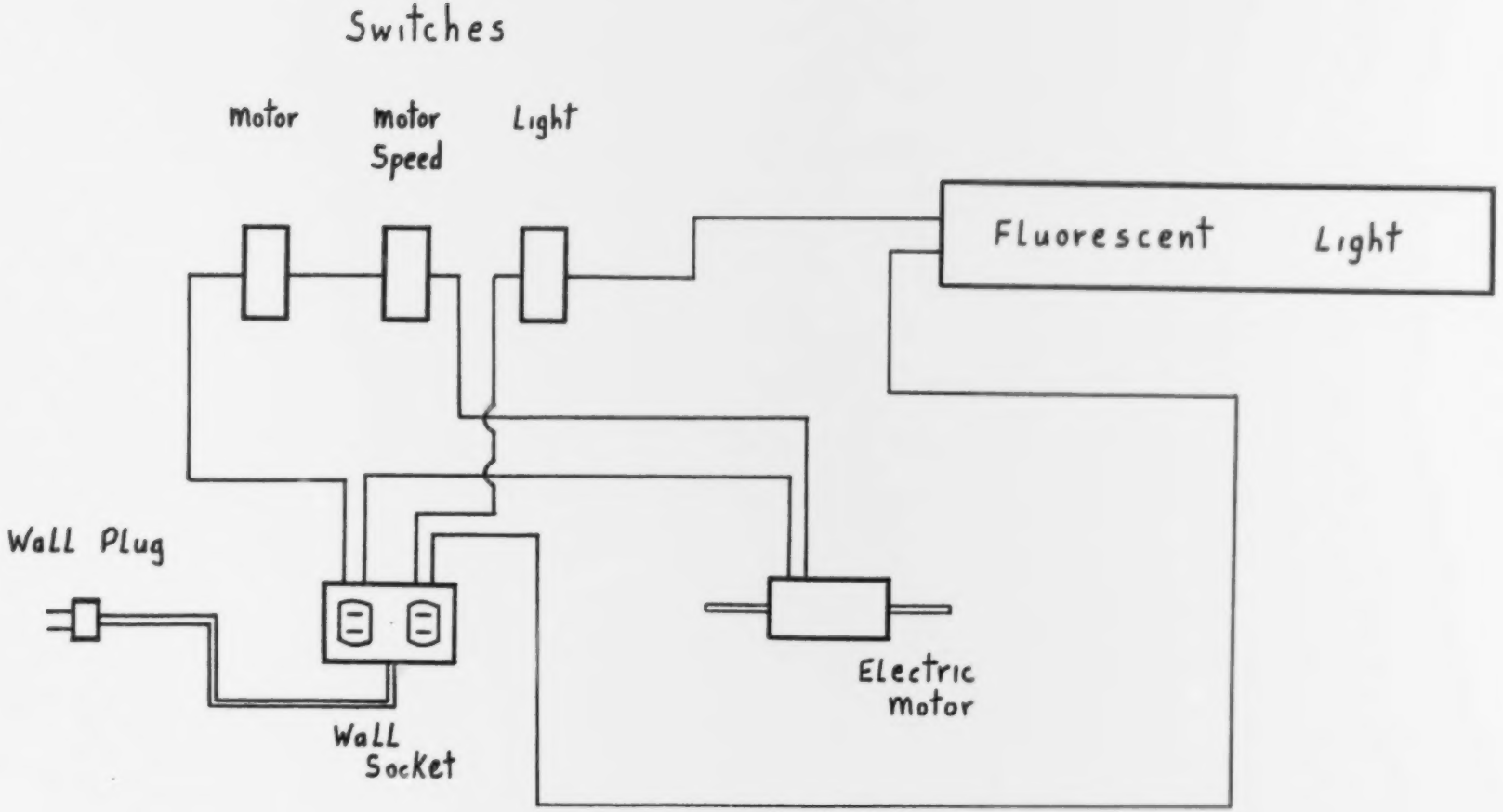
Right End View (w/o Right End-Panel)



Scale: ¼" = 1"

Coupling of Motor Shaft to Threaded Rod





Wiring Diagram of Unit

List of Supplies

1 4' x 8' sheet ½" AC plywood

1 electrical wall socket and cover

2 electrical wall switches and covers

1 electrical dimmer-switch rheostat and cover

1 36" x ¼" threaded rod

1 5000 rpm 1/15 hp electrical motor (AC)

1 18" fluorescent light unit

2 wire letter trays, legal size

1 handle

10 furniture glides

¼ lb. fourpenny finish nails

49 stick-on letters - 1"

10 ft. electrical wire

2 adjustable pipe clamps

1 electrical wall plug

1 qt. paint, flat black

Construction of the rotator unit began with the cutting of the 4' x 8' sheet of plywood into the following dimensions:

Top	44" x 18"
Bottom	44" x 18"
Left end panel	17" x 24"
Center panel	17" x 24"
Right end panel	18" x 24"
Shelf	17" x 23"
Front panel	24" x 24"
Back panel	24" x 24"
Light unit valence	19 ½" x 2"

The left end and center panels, along with the front and back panels, were nailed together forming a box 24 inches long, 18 inches wide, and 24 inches high. The shelf was then mounted inside this box, and the electric motor was installed on the shelf in alignment with a half-inch diameter hole drilled in the center panel of the box. The bottom was then nailed to the box unit, and after the electrical wiring was completed, the top was nailed into place. After the fluorescent light unit was attached to the underside of the top piece, a hole was drilled through the right end panel. When the rotating grid's threaded rod was inserted through this hole and attached to the motor shaft, the right end panel was secured to the top and bottom. The furniture glides were attached to the bottom and the left end panel of the unit, and the handle mounted to the top.

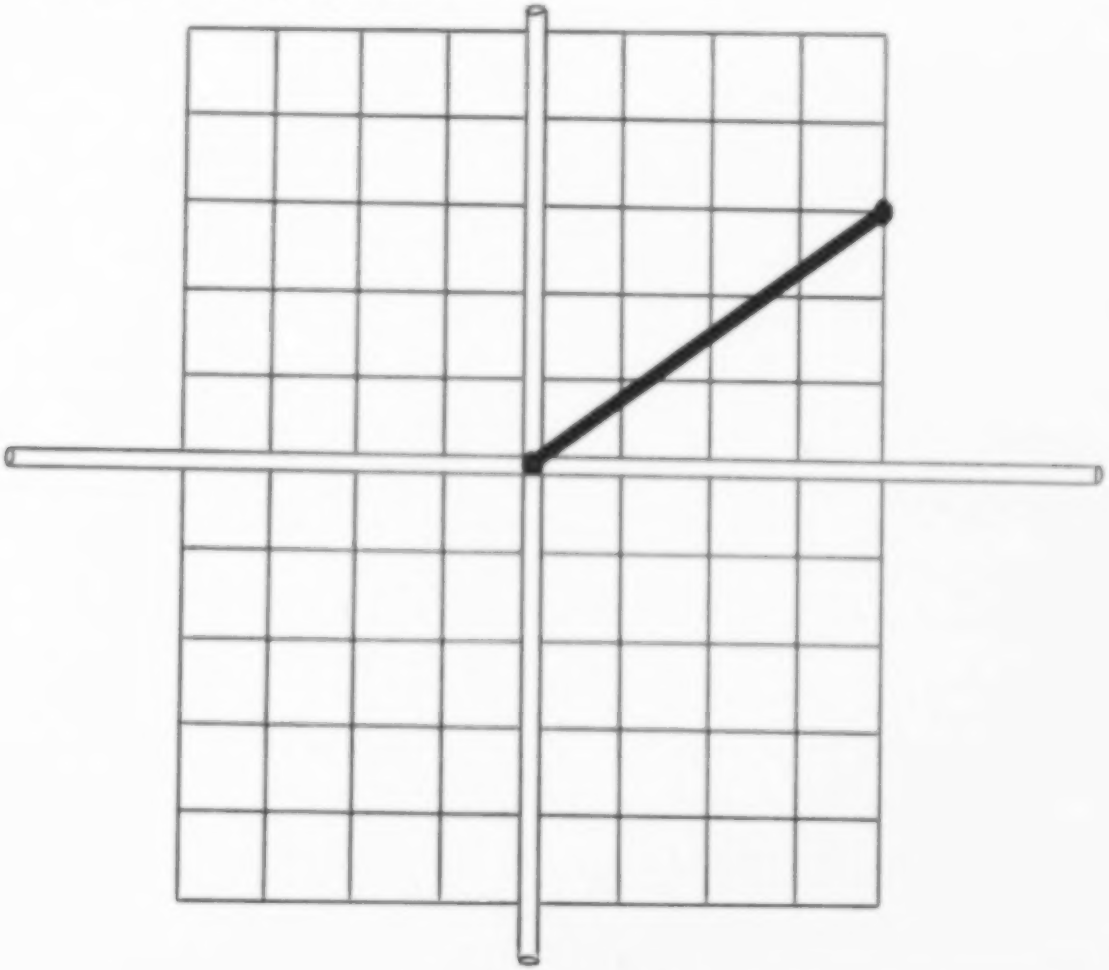
3. Lesson Objectives

1. To insure that the difficult concept of curve rotation is more easily understood from the student's standpoint
2. To provide calculus students with a hands-on activity that will enhance the learning process
3. To increase a student's powers of visualization, thus making easier the transition from abstract concepts to concrete models
4. To show how classroom learning can be found in, as well as applied to, real-world situations
5. To make the class a more enjoyable learning experience

4. Preliminary Background Studies

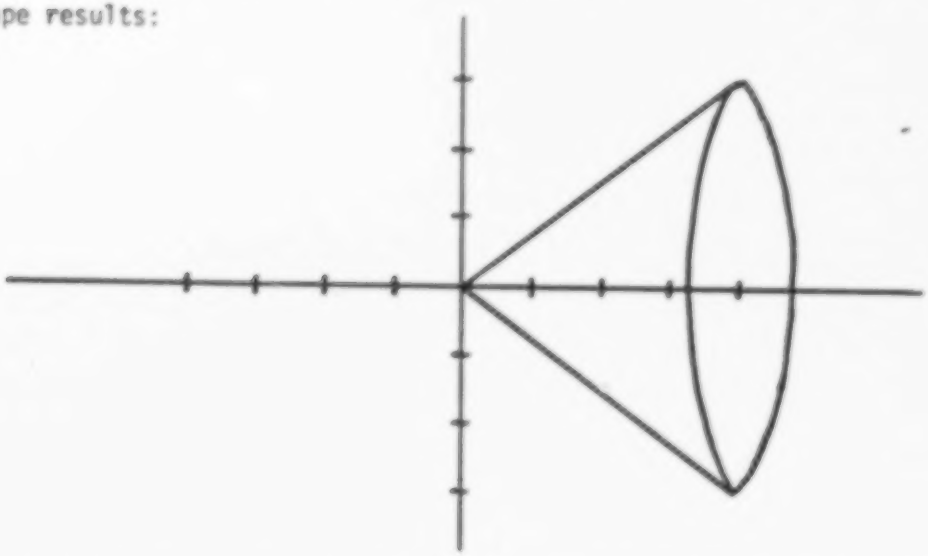
Before the more difficult curve rotation is discussed by me or attempted by my students, I develop the concepts for axial line rotation. When the volume and surface area of line rotation are determined, our results can be compared with, and verified from, facts learned previously in the geometry of a cone, for the Axial Curve Rotator illustrates that two-dimensional lines, on rotation, produce three-dimensional cones. Once these basic techniques are mastered, the students are led into axial curve rotation.

According to the size of the squares in the rotating grid of the device, cut a pipe cleaner approximately six units in length. Wrap its ends around the grid as shown below:

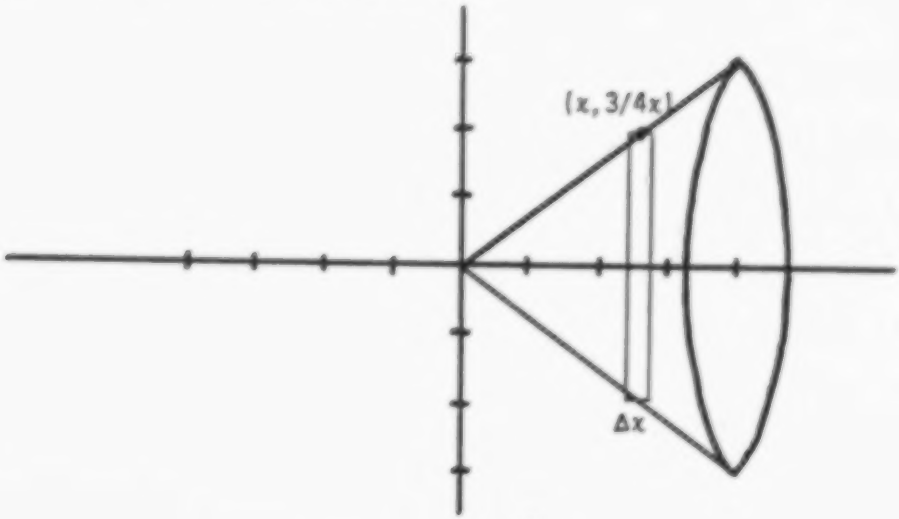


This line segment (5 units in length) shows a rise of 3 units and a run of 4 units. Since it has a slope of 3/4 and a y-intercept of zero, it belongs to the line $y = 3/4x$.

Now consider the linear function $f(x) = 3/4x$, for domain $0 \leq x \leq 4$ and range $0 \leq y \leq 3$. Once the electric motor of the Axial Curve Rotator is engaged, this linear function begins its rotation around the x-axis. As the rheostat steadily increases the rate of rotation, the following shape results:



Using the disk method from calculus, the volume of this three-dimensional shape can be determined as follows:



volume of the disk $= \pi r^2 h$

volume of the disk $= \pi (3/4x)^2 \Delta x$

volume of the disk $= 9/16\pi x^2 \Delta x$

Thus the volume of the line rotation shape can be computed by

$$V = \int_0^4 9/16\pi x^2 \Delta x$$

$$V = 9/16\pi \int_0^4 x^2 \Delta x$$

$$V = 9/16\pi \cdot 1/3x^3 \Big]_0^4$$

$$V = 12\pi \text{ cubic units.}$$

In a similar manner, the surface area of the shape can be computed by

$$\text{surface area} = 2\pi \int_a^b y \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy.$$

Since the line being rotated is represented by $f(x) = y = 3/4x$, it follows that $x = 4/3y$. Then the surface area of the shape becomes

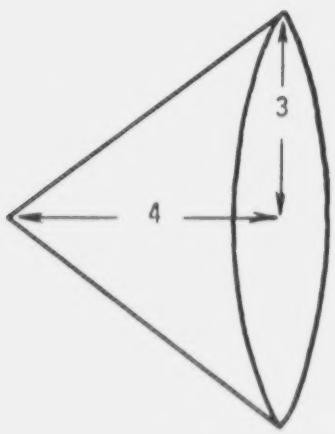
$$A = 2\pi \int_0^3 y \sqrt{1 + (4/3)^2 dy}$$

$$A = 2\pi \int_0^3 y \cdot 5/3 dy$$

$$A = 2\pi \cdot 5/3 \cdot 1/2y^2 \Big]_0^3$$

$$A = 15\pi \text{ square units.}$$

Once the students have mastered the techniques of line rotation using calculus theory to determine both volume and surface area, it should be pointed out that the line being rotated produces the three-dimensional shape of a cone, and the formulas from geometry can be used to check our calculations. The unit from geometry taught the students that the volume of a cone is computed by $1/3\pi r^2 h$ and the surface area by $\pi r\sqrt{r^2 + h^2}$. If the dimensions of our line rotation (cone) are applied to these two formulas, identical results from the calculus method becomes apparent.



$V = 1/3\pi r^2 h$

$V = 1/3\pi \cdot 3^2 \cdot 4$

$V = 12\pi$ cubic units

$A = \pi r\sqrt{r^2 + h^2}$

$A = \pi \cdot 3\sqrt{3^2 + 4^2}$

$A = 15\pi$ square units

Now that each student is familiar with these preliminary background studies in simple line rotation and how their work can be verified with information from a previous course in geometry, we are ready to proceed to curve rotation.

5. Lesson Description

The students will begin their unit on curve rotation by being divided up into two-member work groups. Since the device I constructed has a revolving wire grid containing 1¼-inch blocks, each group of students will be given a sheet of graph paper having blocks of the same size. Each work group will then be assigned an equation to graph on its sheet of graph paper by plotting ordered pairs in the manner learned previously in algebra. These equations will be selected from the following:

	Equation	Domain	Axis to be Rotated
(1)	$y = \frac{1}{2}x^2$	$0 \leq x \leq 3$	x
(2)	$y = \frac{1}{2}x^2$	$0 \leq x \leq 3$	y
(3)	$y = -\frac{1}{2}x^2$	$-3 \leq x \leq 0$	x
(4)	$y = -\frac{1}{2}x^2$	$-3 \leq x \leq 0$	y
(5)	$y = 3 - 1/x$	$1/3 \leq x \leq 4$	x
(6)	$y = 4 - 1/x$	$\frac{1}{4} \leq x \leq 4$	x
(7)	$y = 3\sin 2x$	$0 \leq x \leq \pi/2$	x
(8)	$y = 3\cos x$	$0 \leq x \leq \pi/2$	x
(9)	$y = 3\cos x$	$0 \leq x \leq \pi/2$	y
(10)	$4x^2 + y^2 = 16$	$-2 \leq x \leq 2$	y

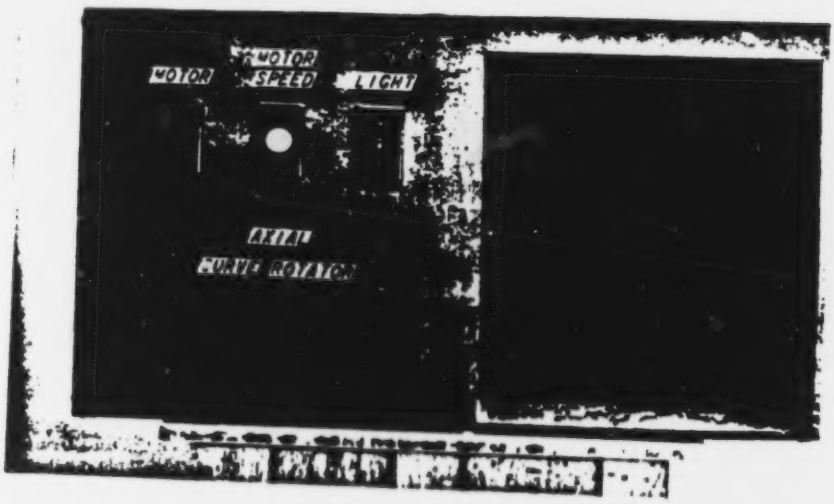
Once its graph is completed, each student group will bend a wire-supported pipe cleaner into the exact shape of its curve on the graph paper. Then, one at a time, each group will attach its pipe cleaner to the grid of the Axial Curve Rotator. When the fluorescent light is turned on and the electric motor engaged, thereby revolving the grid, the students will observe their two-dimensional curve changing into a

three-dimensional shape, right before their eyes. After observing similar procedures conducted by the other class members, students will begin developing the necessary mental skills that will enable them to visualize and sketch algebraic curves being rotated about either axis of the coordinate plane. After the mechanical curve rotation phase of the lesson is completed, students will sketch the resulting three-dimensional shape in their notebook. Once this task is completed, they will be instructed to sketch a representative disk perpendicular to the axis of rotation. Then by using the line rotation method learned in section 4, students will calculate the volume and surface area of the solid of rotation over the domain used.

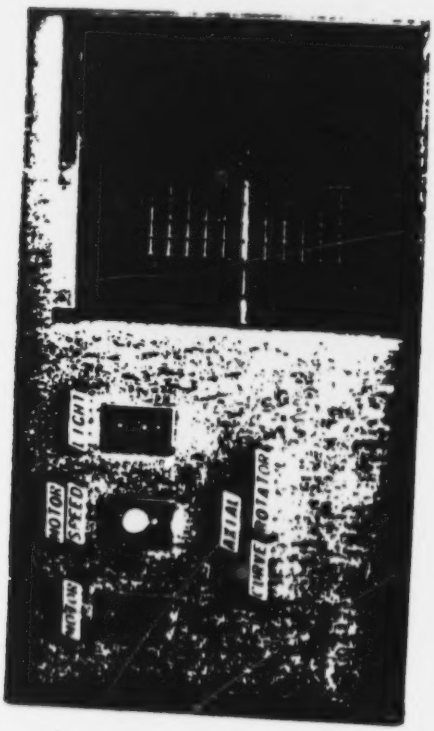
The visualization skills gained during this exercise will be of great benefit in upcoming chapters dealing with the moment of inertia of solids resulting from curves in rotation around an axis. Also, the force exerted by liquids onto the face of a semicircular or parabolic plate can be computed and visualized by rotating the necessary curve around the y-axis of the Axial Curve Rotator. And finally, the work done in pumping water to certain heights of various vessels can be determined with greater ease after the student has viewed the three-dimensional shape being used on my curve rotator device.

It is therefore obvious to me that the Axial Curve Rotator has brought to life a difficult concept from calculus. The abstract concept of curve rotation has now been changed to a topic with concrete models that the student can see firsthand and use in his or her study of the calculus.

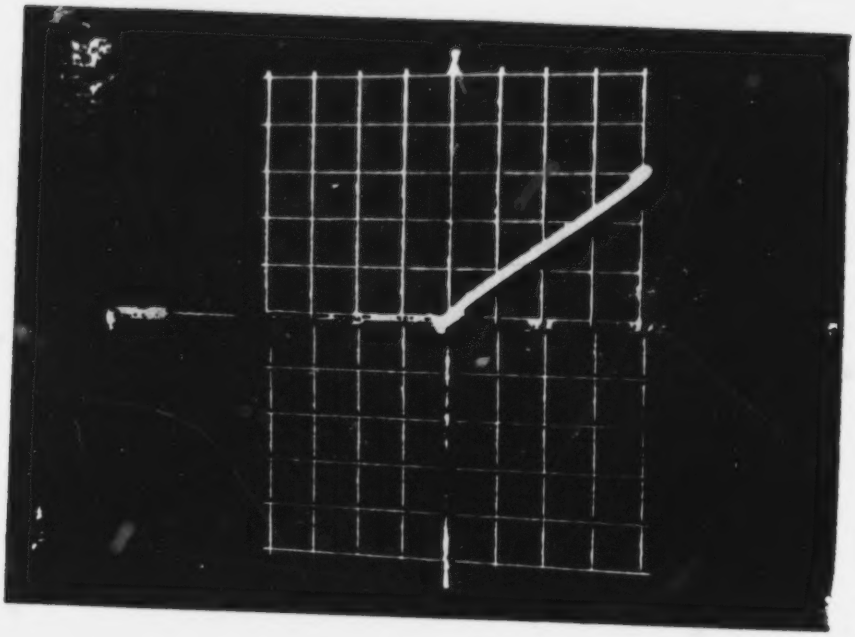
6. Photographs of Unit with Descriptions



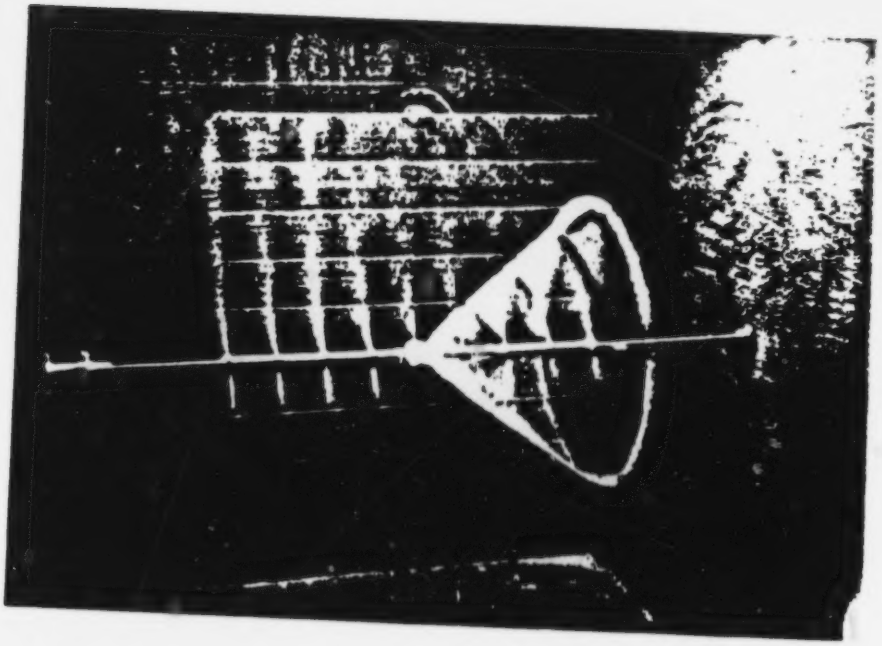
The Axial Curve Rotator positioned for x-axis rotation



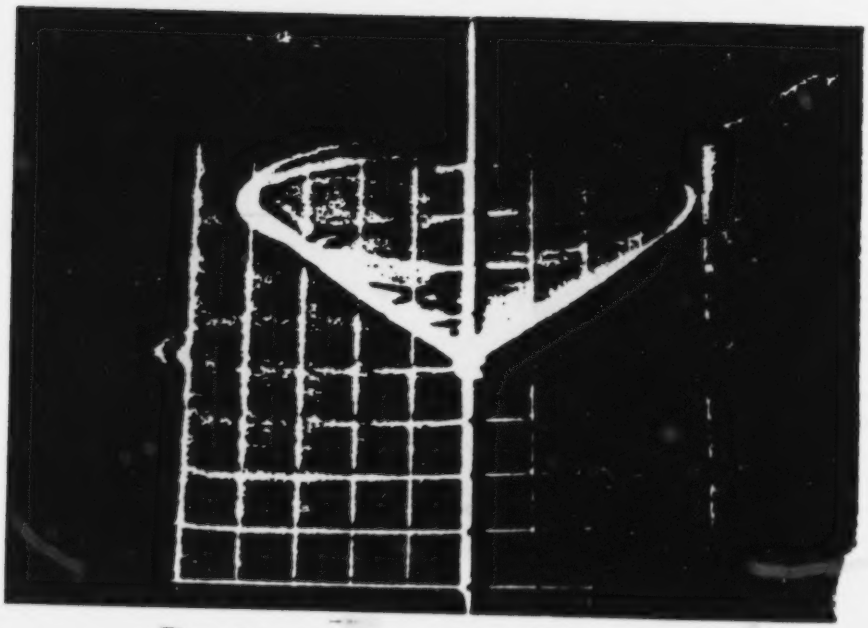
The Axial Curve Rotator positioned for y-axis rotation



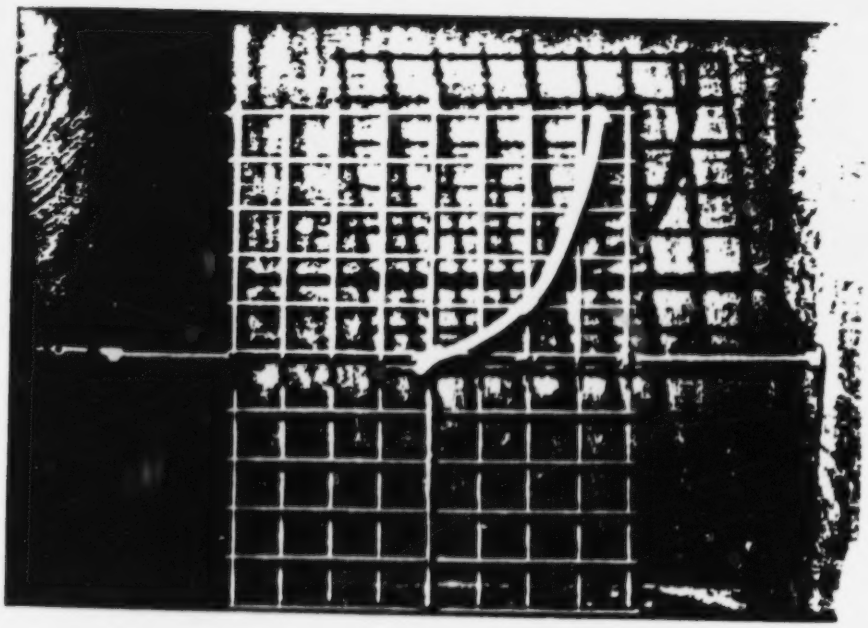
Pipe cleaner on wire grid illustrating the equation $y = 3/4x$ for $0 \leq x \leq 4$



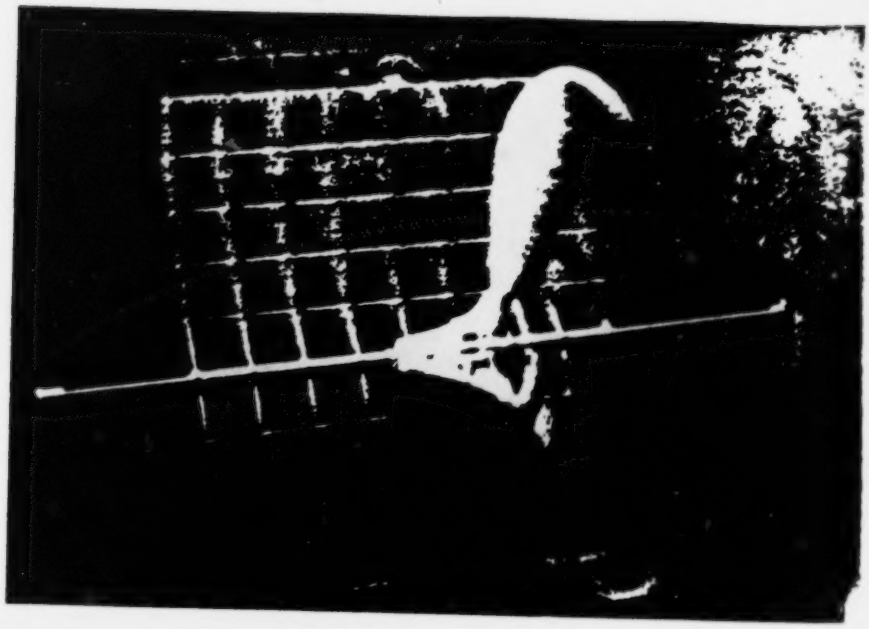
The equation $y = 3/4x$ during x-axis rotation, thus demonstrating that line rotation produces a cone



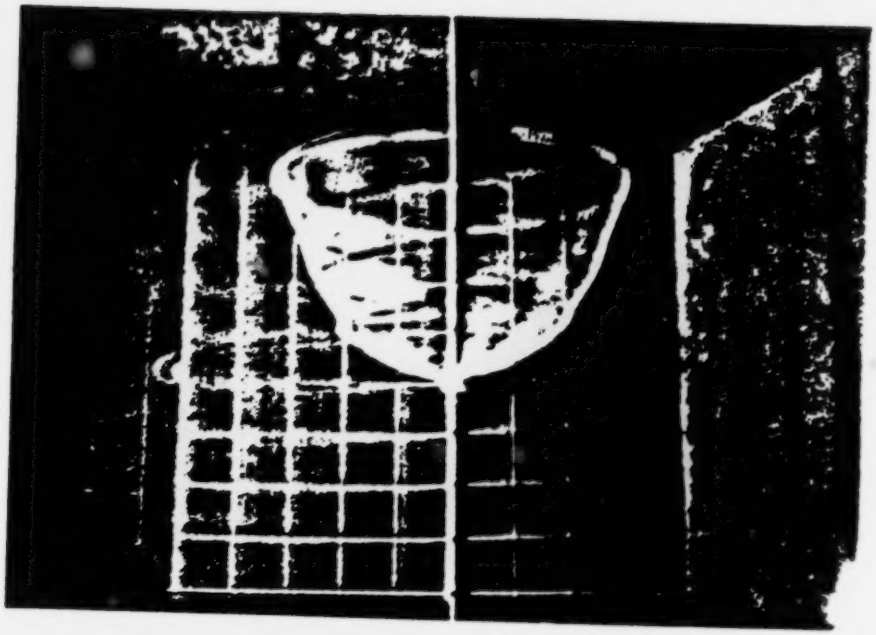
The same equation $y = 3/4x$ under y-axis rotation



Pipe cleaner on wire grid illustrating the equation $y = 1/8x^3$ for $0 \leq x \leq 3\frac{1}{2}$



The equation $y = 1/8x^3$ during x-axis rotation



The same equation $y = 1/8x^3$ under y-axis rotation

7. Real-World Applications of Teaching Unit

Objects constructed using line rotation and curve rotation shapes are found all around us. They can be found within the home, in the structures in which we work and live, in the products of the manufacturing industry, and in the utilities that bring us convenience and enjoyment, as well as in science and technology.

Around the home, items using these shapes include colanders and funnels for draining liquids in the kitchen, as well as sherbert dishes and reamers. Also included are makeup mirrors having a magnifying concave surface, outdoor lamp fixtures for exterior lighting convenience, and conical-shaped chimney caps for ventilation purposes.

Products of line and curve rotation also show up in the construction industry, which builds our homes and offices. Surface area of rotation is of great importance in determining the amount of roofing material necessary to cover many structures found in our society. Examples of these include Victorian-styled homes with their turrets and backyard gazebos.

The many shapes of line and curve rotation also have industrial applications. The volume methods of line and curve rotation are necessary in determining the quantity of grain, water, and so on, that can be stored in tanks of various shapes. Industrial examples include grain storage silos and water tank roofs using line rotation cones whose bases have curve rotation hemispherical shapes.

The shapes of curve rotation are also found in objects that make our lives more enjoyable. The parabolic reflectors of the communication industry are found in abundance dotting our countryside. This shape is found in backyard satellite receiving dishes for television viewing and microwave transmission dishes to help us communicate.

In science and technology, the aerodynamic shapes from line and curve rotation are well suited for the leading edge of air and space vehicles. The nose cones of airplanes, rockets, and reentry vehicles make use of the shapes that the Axial Curve Rotator can produce in the classroom. The amount of material necessary for their construction can be derived from the surface area formula learned in calculus class. Examples of this shape found in aviation include the nose of jet airplanes as well as the jet fuel tanks. The nose cones of the space shuttle's solid rocket boosters and external fuel tank also illustrate line and curve rotation shapes.

Thus it is apparent that the study of line and curve rotation in calculus plays an important part in the construction of many real-world objects.